Så, som i exempel 4.2.2 har vi:

$$\frac{1}{\sqrt{4}}\begin{bmatrix} | & | & | & | \\ \underline{X}^0 & \underline{X}^1 & \underline{X}^2 & \underline{X}^3 \\ | & | & | & | \end{bmatrix}\begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\frac{1}{2}\begin{bmatrix} (1)^0 & (1)^1 & (1)^2 & (1)^3 \\ (i)^0 & (i)^1 & (i)^2 & (i)^3 \\ (-1)^0 & (-1)^1 & (-1)^2 & (-1)^3 \\ (-i)^0 & (-i)^1 & (-i)^2 & (-i)^3 \end{bmatrix}\begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} .1 \\ .3 \\ .5 \\ .7 \end{bmatrix}$$

$$\underbrace{\frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix}}_{B_4}\begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} .1 \\ .3 \\ .5 \\ .7 \end{bmatrix}$$

Lösa med Gausselimination

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & .2 \\ 1 & i & -1 & -i & .6 \\ 1 & -1 & 1 & -1 & 1 \\ 1 & -i & -1 & i & 1.4 \end{array}\right] \begin{array}{c} (-1) \\ \\ \\ \\ \end{array} \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & .2 \\ 0 & i-1 & -2 & -i-1 & .4 \\ 0 & -2 & 0 & -2 & .8 \\ 0 & -i-1 & -2 & i-1 & 1.2 \end{array}\right] \begin{array}{c} \\ (1) \\ \\ \\ \end{array} \sim$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & .2 \\ 0 & i-1 & -2 & -i-1 & .4 \\ 0 & -2 & 0 & -2 & .8 \\ 0 & -2 & -4 & -2 & 1.6 \end{array}\right] \begin{array}{c} \\ \\ (-1) \\ \\ \end{array} \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & .2 \\ 0 & i-1 & -2 & -i-1 & .4 \\ 0 & -2 & 0 & -2 & .8 \\ 0 & 0 & -4 & 0 & .8 \end{array}\right] \sim$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & .2 \\ 0 & i-1 & -2 & -i-1 & .4 \\ 0 & 0 & -4 & 0 & .8 \\ 0 & -2 & 0 & -2 & .8 \end{array}\right) \quad (-i-1) \sim \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & .2 \\ 0 & 2 & 2i+2 & (-i-1)^2 & -.4i-.4 \\ 0 & 0 & -4 & 0 & .8 \\ 0 & -2 & 0 & -2 & .8 \end{array}\right) \quad (1) \sim$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & .2 \\ 0 & 2 & 2i+2 & (-i-1)^2 & -.4i-.4 \\ 0 & 0 & -4 & 0 & .8 \\ 0 & 0 & 2i+2 & -2+(-i-1)^2 & -.4i+.4 \end{array}\right) \quad \frac{1}{2}(i+1) \sim$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & .2 \\ 0 & 2 & 2i+2 & (-i-1)^2 & -.4i-.4 \\ 0 & 0 & -4 & 0 & .8 \\ 0 & 0 & 0 & -2+(-i-1)^2 & .8 \end{array}\right)$$

<u>så har vi:</u>

$$y_0 + y_1 + y_2 + y_3 = .2$$
$$2y_1 + (2i+2)y_2 + (-i-1)^2 y_3 = -.4i-.4$$
$$-4y_2 = .8$$
$$\left(-2+(-i-1)^2\right)y_3 = .8$$

Fjärde linje ger:

$$\left[-2+\left(i^2+2i+1\right)\right]y_3 = .8$$
$$\left[-2+\left(-1+2i+1\right)\right]y_3 = .8$$
$$\left(-2+2i\right)y_3 = .8 \implies y_3 = \frac{.8}{(-2+2i)}$$

$$y_3 = \frac{.8}{(2+2i)} \cdot \frac{(-2-2i)}{(-2-2i)} = \frac{-1.6-1.6i}{8} = -.2-.2i$$

Tredje linje ger: $y_2 = \frac{-.8}{4} = -.2$

Andra linje ger:

$$2y_1 + (2i+2)y_2 + (-i-1)^2 y_3 = -.4i - .4$$

$$2y_1 + (2i+2)(-.2) + (2i)(-.2-.2i) = -.4i-.4$$

$$2y_1 - .4i - .4 - .4i + .4 = -.4i - .4$$

$$2y_1 = .4i - .4$$

$$y_1 = .2i - .2$$

Första linje ger:

$$y_0 + .2i - .2 - .2 - .2 - .2i = .2$$

$$y_0 = .8$$

Svaret:

$$y = \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} .8 \\ -.2+.2i \\ -.2 \\ -.2-.2i \end{bmatrix}$$

4.6

$$x = \begin{bmatrix} .1 \\ .3 \\ .5 \\ .7 \end{bmatrix} \qquad B_4 = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix}$$

Vi vill lösa

$$B_4 \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix},$$

$$\frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix}\begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} .1 \\ .3 \\ .5 \\ .7 \end{bmatrix} \qquad (1)$$

Skapa $F_n := B_n^{-1} = \overline{B}_n$

$$F_4 = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{bmatrix}$$

och multiplicera (1) från den vänstra sidan med F_4:

Kom ihåg!

$F_4 \cdot B_4 = I$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{bmatrix}\begin{bmatrix} .1 \\ .3 \\ .5 \\ .7 \end{bmatrix}$$

$$\begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} .1 + .3 + .5 + .7 \\ .1 - .3i - .5 + .7i \\ .1 - .3 + .5 - .7 \\ .1 + .3i - .5 - .7i \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1.6 \\ -.4 + .4i \\ -.4 \\ -.4 - .4i \end{bmatrix}$$

$$= \begin{bmatrix} .8 \\ -.2 + .2i \\ -.2 \\ -.2 - .2i \end{bmatrix}$$