Assignment 2 – by Kathlén Kohn
Commutative Algebra and Algebraic Geometry

- Course code: MM7042 (at Stockholm University).
- This course is given jointly by Stockholm University and KTH, and part of the course is given at KTH.
- Number of credits: 7.5.
- Number of students: ~30.
- Number of lecturers: 2 (one from Stockholm University, one from KTH).

Course content
Algebraic geometry is the study of solutions to systems of polynomial equations. Commutative algebra is the underlying machinery. The course will give an introduction to these areas.

The course covers rings, ideals, prime ideals, nilpotents, zero-divisors, modules, Noetherian rings, Hilbert's basis theorem, finite extensions and Noetherian normalization, varieties, Nullstellensatz, prime ideal spectra, localization, primary decomposition.

Learning objectives
After the course the student should be able to
- apply theorems and methods within the topic of the course,
- formulate central definitions and theorems within the topic of the course,
- describe and formulate basic proofs within the topic of the course,

Rating scale
A, B, C, D, E, Fx, F

Examination
- HOM1 – homework assignments, 5.0 hp, rating scale: A, B, C, D, E, Fx, F
- TEN1 – oral examination, 2.5 hp, rating scale: P, F

The examiner decides, in consultation with KTHs Coordinator of students with disabilities (Funka), about any customized examination for students with documented, lasting disability.

The examiner may allow another form of examination for re-examination of individual students.

Goal-related grading criteria

Objective	E	D	C	B	A
apply theorems and methods within the topic of the course	to basic problems	to advanced problems within *some* part of the course	to advanced problems within *several* parts of the course	[C criterion] + to advanced problems within *all* parts of the course, **or** solve problems combining several parts of the course, **or** generalize theorems and methods within the topic of the course	to advanced problems within *all* parts of the course, **and** solve problems combining several parts of the course, **and** generalize theorems and methods within the topic of the course
	assessed using homework assignments HOM1				
formulate central definitions and theorems within the topic of the course assessed using oral examination TEN1					
describe and formulate basic proofs within the topic of the course assessed using oral examination TEN1					

Table 1: grading criteria

Final grade

To pass the course, TEN1 has to be passed and at least grade E needs be reached in HOM1. Then the final grade is the grade reached in HOM1.

Examination details

HOM1 – homework assignments

There will be 8 homework assignments that ask the students to solve problems and hand in their written solutions. These are individual assignments. To ensure that each student solved the problems of an assignment by themselves, a short oral test will be scheduled a few days after the deadline, where the student explains their solutions to a TA or lecturer.

The homework assignments contain 4 types of problems (see Table 1 for an explanation):

- B – basic
- A – advanced
- G – generalization
- C – combination

Each homework assignment contains 3 problems, each problem giving 5 points:

1. B problem
2. A problem
3. A/G/C problem (i.e., a problem either of type A, G or C)

To reach a grade, the student has to obtain the following amount of points:

- Fx: 25 pts
- E: 30 pts
- D: 45 pts

- C: 60 pts
- B: 75 pts, whereof:
  - either 5 points in total for problems of type G and C,
  - or a total of 5 points in *every* homework assignment for problems of type A, G and C
- A: 90 pts, whereof 5 points in G problems, 5 points in C problems, and 5 points in *every* homework assignment for problems of type A, G and C

A student who reached grade Fx can obtain grade E by achieving 11 points out of 15 total in a special homework assignment consisting of 3 problems of type B.

Note that at most 40 points can be reached with basic problems. Hence, to reach grade E at least 75% of all basic problems need to be solved (or compensated with A/G/C problems). In particular, students reaching grade E fulfil the learning objective *to apply theorems and methods within the topic of the course to basic problems*. Moreover, the point requirement for grade D ensures that at least 5 points are obtained in A/G/C problems. Similarly, all point requirements listed above ensure that this assessment is aligned with the grading criteria in Table 1.

TEN1 – oral examination
The oral exam takes 25 minutes and is held by both lecturers.

In the first 5 minutes of the exam, the student will present a randomly assigned topic from the course. The student will be informed about their topic via email at least 1 hour before their exam. The student can freely choose which definitions, theorems, and proofs within the topic to highlight in their presentation. Afterwards, the lecturers might ask clarifying question on the topic.

In the remaining 15-20 minutes of the exam, the lecturers will ask the student to formulate definitions, theorems, and basic proofs from at least two other topics treated in the course.

To pass the oral examination, the student’s presentation should describe central definitions, theorems, or proofs within the randomly assigned topic and the student should be able to answer all but one question by the lecturers. This ensures that the students passing the oral examination fulfil the learning objectives *to formulate central definitions and theorems within the topic of the course* and *to describe and formulate basic proofs within the topic of the course.*

Reflection

ILOs

I aimed to formulate the ILOs such that they are assessable, result oriented, and at a reasonable level (i.e. such that every passing student fulfils them). The 2nd and 3rd ILO only need to be assessed at E-level since they require the students to reproduce definitions, theorems, and basic proofs that they learned in the class. A student can either do that or not, so it does not make sense to assess this skill at different levels. The 1st ILO should be assessed at levels A to E. For this, the difficulty and variety of the problems that are supposed to be solved with methods and theorems from the course should distinguish the levels. The grade of execution (i.e., quality or niceness of a problem solution) should not influence the student's grade as this is a highly subjective matter of taste.

Grading criteria

I distinguish 3 difficulty levels in the problems to be solved to fulfil the grading criteria for the 1st ILO: 1) basic problems, 2) advanced problems, 3) generalization / combination problems. I expect a grade-A student to be able to solve advanced within all topics of the course and to also solve some problems of the highest difficulty level, where the students are either asked to combine several parts of the course to solve a problem or to generalize a theorem / method from the course to a new or larger context. If a student can solve advanced problems within all topics of the course, but does not succeed with problems on the highest difficulty level, I consider the student as a grade-B student. It is acceptable to miss problems from some topics of the course and compensate that by solving problems on the highest difficulty level instead, to still receive grade B. However, if a student is not able to do this compensation, they become a grade-C student. As grade-E students are only supposed to be able to solve basic problems and grade-D students should be in between grade-E and grade-C students, a grade-D student should be able to solve at least one advanced problem.

Assessment

I decided to use two different forms of assessment (written homework assignments vs. oral exam) to test different skills of the students, as some might naturally excel at written and others at oral presentation.

The reproduction of theorems, definitions, and proofs tests the basic knowledge of the student, which is a very different skill than the application (and generalization) of theorems and methods. Hence, I decided to assess the 2nd and 3rd ILO in the final oral exam using a pass-fail grading scale. Moreover, I decided to begin the oral examination with a presentation of a randomly assigned topic that the student could prepare in advance, to make the entrance into the oral exam less intimidating and give the student a chance to have a good and self-directed start. After this initial phase, I decided that the lecturers should ask questions about definitions, theorems, and proofs from at least two different topics of the course (also different from the randomly assigned topic) to ensure that the students have studied (almost) all topics for the exam. Finally, I believe that a duration of 25 minutes gives enough time to talk, without stress, about in total 3 or more topics from the course. At the same time, 25 minutes are not too long to be overly intimidating.

I decided to assess the 1st ILO in the homework assignments (incl. oral tests). Let me motivate that decision a little further: First, I believe that homework assignments are a reasonable form for

assessing the 1st ILO since it gives the students enough time to also solve advanced problems. Second, regular homework makes sure that the students study throughout the term instead of only for the final exam. Providing 8 homework assignments means that the students will have to hand in written solutions every other week. Third, since there is a risk that the students cooperate (although they are not allowed to) when producing written solutions for the homework problems, one can check the students' understanding in an oral test where they have to explain their solutions to a TA or lecturer. Finally, to assess the different grading criteria of the 3rd ILO, problems of different levels of difficulty will be posed during the homework assignments. As described in the report, the distribution of point requirements to reach grades A – E ensures that this assessment is aligned with the grading criteria in Table 1.

Feedback

From my peers in the class, I got mainly positive feedback on my first submission. I received only one piece of critical advice: One peer did not agree with my change from the original ILOs of the course to remove "generalize theorems and methods" as an ILO on its own and to add it to another existing ILO ("applying theorems and methods") for the higher-grade criteria. They felt that this change made the discrepancy between ILOs and grade-A criteria too large. However, after thorough discussions with two students as well as my discussion partner during the last class, I decided to stay with my change because I think that "generalizing theorems and methods" is a too advanced skill to be required for all students passing the course. The peer who provided the feedback agreed to that way of thinking, but suggested to at least increase the difficulty of the ILOs since they felt that my ILOs sound too much like ILOs for a 1st cycle course (although my course is a 2nd cycle course). Also here, after discussing with 2 students, I decided to keep my ILOs as they generally describe what is expected of students in almost any course in mathematics, no matter if it is a 1st, 2nd, or 3rd cycle course.

I asked two students, who took the course when I taught it last fall, for feedback on my first submission. Afterwards, I also discussed with both of them. Both students criticized that last fall we required the students to solve problems in the final oral examination, which can be very difficult for the students depending on their level of nervosity. Moreover, both students disliked that last fall the oral examination gave more points for the final grade than the continuous homework assignments, as this puts a lot of pressure on the final performance. I agree with the students regarding both points. Hence, I implemented the following two changes in this report: First, problem solving skills are now only checked in homework assignments. Secondly, the oral examination uses the pass-fail grading scale to reduce the pressure. In the new version of this report, both students like the idea of continuous examination using homework assignments.

Each student had an additional remark. Despite the change to pass-fail grading on the oral exam, one student still finds the oral exam so scary that they are worried that it "can scare some students away since it is a very stressful situation and you are as a person being forced to 'expose yourself'." Instead the student suggested to have a written exam to assess the 2nd and 3rd ILO. I definitely see the point of the student, but I also believe that 1) an oral examination is a great learning experience, 2) formulating proofs (see 3rd ILO) correctly in a written exam is harder than to explain the proof idea orally, 3) having a written exam would make all assessments to be written and hence reduce the variety of assessments forms, and 4) I aim to make the oral exam as little intimidating as possible

by having the students present a topic they could prepare in advance at the beginning of the exam. So all in all, I decided to stay with a final oral exam as described in the report.

The other student is sceptical about the short oral tests after each homework assignment that serve as quality assurance of the grading. They pointed out that these tests do not really hinder students from cheating by collaborating as long as they are able to understand the problem solutions afterward. However, I generally trust that almost all students won't cheat in the first place. The short oral tests have a similar spirit as Zoom surveillance during online exams: these measures are there to underline that cheating is not allowed, to not motivate an unusual amount of student to try to cheat. Moreover, my experience is that it is rarely the case that students fully understand the solutions by other students such that they can explain them perfectly, which should almost always allow the lecturer or TA to identify cheating students.

Using grading criteria while teaching

During the lectures, the lecturer should clearly highlight the definitions and theorems such that the students are aware which concepts they should know in the oral exam. Similarly, the lecturer should identify which proofs from the lectures or course material are considered basic. This can be done either during the lectures, in lecture notes, or after each lecture on the course website.

In addition to the lectures, the course has a weekly exercise session held by a TA. This is the perfect opportunity to prepare the students for the homework assignments. The exercise sessions should discuss examples and problems that both help the students to understand the course material better and train them to be able to solve the homework problems. The exercise session also is an opportunity for the students to discuss examples and problems in smaller groups such that they can learn from each other. The problems discussed in the exercise sessions should have varying difficulty to reflect both the different skill levels of the students and the range of homework problems according to the grading criteria in Table 1.

Effects of combining the grades

The grading criteria and assessment should put the main focus of the students on the homework assignments (instead of the oral exam). This is intended, as the students who work on all homework problems throughout the course should be able to pass the oral exam without additional long study sessions. In particular, I expect the students to learn continuously during the course as they have to hand in solutions every other week.

The fact that the grades only depend on the homework assignments could motivate more students to cheat, e.g. by collaborating on the homework problems. The purpose of the oral tests is to identify such relations among students. Another issue that might arise is that students who have not focused on the first few homework assignments could tend to give up on the 2nd half of the course, as they cannot reach their desired grade anymore. The lecturer could identify if several students feel like that using a mid-term course evaluation. To address this issue, if it occurs, the lecturer could add bonus questions (on previous topics) to some homework assignments to show the students that they still have a chance to keep up which can re-kindle their motivation for the course.