Grading Criteria for FSF3945: Advanced Probability

Credits: 7.5 ECTS             Number of students: About 8 to 10.

1 Intended Learning Outcomes

After passing the course, the students should be able to

- Explain basic definitions and important theorems within advanced probability.
- Describe basic proofs within advanced probability.
- Construct examples and counterexamples within advanced probability.
- Apply theorems to solve problems within advanced probability.
- Present written solutions to problems within advanced probability in a mathematically rigorous way.

2 Course Content

This course consists of seven bi-weekly meetings and a homework is assigned after each meeting. Each meeting (except the first one) contains a one-hour homework discussion session and a one-hour lecture for a new topic. During the discussion session, some of the students are designated (one week prior to the scheduled meeting time) to present their solutions of certain problems in the homework which is given after previous meeting. The lecture comes after the discussion session. There will be a two-hour office hour every week. Additionally, the topic for each meeting is given below:

<table>
<thead>
<tr>
<th>Topics</th>
<th>Content</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Random walks and the heat equation</td>
<td>Simple random walk; Heat equation; Escape time</td>
</tr>
<tr>
<td>2 Infinite divisibility</td>
<td>Infinitely divisible distributions; Lévy process; Additive process</td>
</tr>
<tr>
<td>3 Large deviations</td>
<td>Large deviation principle; Laplace principle; Legendre transform; Contraction principle</td>
</tr>
<tr>
<td>4 Weak convergence I</td>
<td>Weak convergence of probability measures; Portmanteau theorem; Continuous mapping theorem; Tightness; Prohorov’s theorem</td>
</tr>
<tr>
<td>5 Weak convergence II</td>
<td>Donsker’s theorem; Existence of Brownian motion; Tightness in $C[0, 1]$-space</td>
</tr>
<tr>
<td>6 Brownian motion</td>
<td>Reflection principle; Properties of Brownian motion</td>
</tr>
<tr>
<td>7 Ergodic theory</td>
<td>Measure-preserving transform; Birkhoff-Khinchin theorem; Von Neumann’s mean ergodic theorem</td>
</tr>
</tbody>
</table>
Remark 2.1 Notice that this is a non-standard graduate course in the sense that the students are expected to do lots of self-reading and self-learning after each lecture. To be more precise, the instructor will only explain a bigger picture (for instance, why certain topic is important) of each topic without going into details of some results (for example, only sketch of proofs will be given in lectures).

3 ILOs and Assessment

The assessment tasks of the course are 1. seven homework assignments, 2. some number (depending on the number of registered students) of presentations of homework solutions and 3. a final oral exam. The relation between ILOs and assessments is displayed in the following table.

<table>
<thead>
<tr>
<th>Intended learning outcome</th>
<th>Homework</th>
<th>Presentation</th>
<th>Oral exam</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explain basic definitions and important theorems within advanced probability</td>
<td></td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>Describe basic proofs within advanced probability</td>
<td></td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>Construct examples and counterexamples within advanced probability</td>
<td>x</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>Apply theorems to solve problems within advanced probability</td>
<td>x</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>Present written solutions to problems within advanced probability in a mathematically rigorous way</td>
<td></td>
<td></td>
<td>x</td>
</tr>
</tbody>
</table>

4 Grading Criteria

Grading levels: F(fail), P(pass) and PwD(pass with distinction).

<table>
<thead>
<tr>
<th>Intended learning outcome</th>
<th>P</th>
<th>PwD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explain basic definitions and important theorems within advanced probability</td>
<td>with some minor errors</td>
<td>with almost no error</td>
</tr>
<tr>
<td>Describe basic proofs within advanced probability</td>
<td>with some hints or minor errors</td>
<td>without a hint and with almost no error</td>
</tr>
<tr>
<td>Construct examples and counterexamples within advanced probability</td>
<td>with some hints or minor errors</td>
<td>without a hint and with almost no error</td>
</tr>
<tr>
<td>Apply theorems to solve problems within advanced probability</td>
<td>to solve basic problems</td>
<td>to solve advanced problems</td>
</tr>
<tr>
<td>Present written solutions to problems within advanced probability in a mathematically rigorous way</td>
<td>ILO itself</td>
<td></td>
</tr>
</tbody>
</table>

Remark 4.1 ILO 5 is assessed by pass level only.
5 Assessment and Final grade

5.1 Homework

There are seven homework problem sets which are given after each lecture. Each homework contains 5 basic problems and 2 advanced problems. Submission of advanced problems is not mandatory. As for the grading of each homework, every problem is graded by 3 points. If the student’s answer is almost flawless, then 3 points is given. If the answer is correct with some minor errors or with less detailed explanations, then 2 points is given. If the answer is far away from the correct answer, then only 1 point is assigned. If the student leaves the answer blank, then they receive no point. Therefore, I strongly encourage everyone to at least give it a try!

To get a pass in a given homework, one has to receive at least 9 points out of 15 points from the basic problems. To get a PwD, one has to achieve the P level and receive at least 4 points out of 6 points from the advanced problems.

Remark 5.1 ILO3 and ILO4 are assessed in every homework.

Remark 5.2 Late homework would be considered as a fail unless an approval is given a priori.

Remark 5.3 If a student obtains an F in a homework assignment, then they can resubmit within a week to get a second chance. However, since skeleton solutions will be published within one day after the submission deadline, a pass for a re-submission requires 12 points out of 15 points and no PwD will be given.

5.2 Presentation

All registered students will be assigned to present their solutions for homework problems alternatively in homework discussion sessions. There will be an office hour every week, so the students are encouraged to attend if they have any difficulty with solving the assigned problem or with understanding any material of the course. A pass in a presentation is given if the presentation of a solution is mathematically rigorous and understandable.

Remark 5.4 ILO5 is assessed in every presentation.

5.3 Oral exam

The oral exam comprises two parts:

- 10 minutes summary of a topic which is chosen randomly from all the seven topics in the course content (ILO1 and ILO2 are assessed during this part).

- 20 minutes of Q&A (ILO3 and ILO4 are assessed during this part).

The first part is assessed by P and PwD. One can receive P for the first part if the presentation is highly relevant to the chosen topic and covers most of the corresponding main contents, and there are only some minor errors in the presentation. To receive PwD for the first part, the presentation has to contain almost no error. As for the Q&A, it has the same structure as homework assignments. Namely, there are 5 basic problems and 2 advanced problems. Every problem is also evaluated by 3 points with the same standard described before. However, the difficulty of the problems in the oral exam are expected to be easier than in homework so
that it is possible to answer on spot. Examples of possible problems in the oral exam will be
given during the course.

Moreover, to get a pass in the Q&A, one has to receive at least 10 points out of 15 points
from the basic problems. To get a PwD, one has to achieve the P level and receive at least
4 points out of 6 points from the advanced problems. A reminder will be given when the
student is assigned with an advanced problem, so that the student would not be too worried
about failing the course if they can not answer the question.

5.4 Final grade

*Final grade scales:* F(fail), Fx, P(pass) and PwD(pass with distinction).

**Remark 5.5** In reality, since it is a graduate course for PhD students, only pass or fail is
given in the end. Nevertheless, I would implement my grading according to three levels F,P
and PwD. At the end of the course, those students who pass the course can know if they receive
P or PwD upon request. I think this might motivate some students to learn more.

The following table summarizes how the grades from different assessments are combined into
one final grade.

<table>
<thead>
<tr>
<th>Final grade</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Fx</strong></td>
<td>Pass on EVERY homework, and EVERY presentation, but fail in ONLY ONE PART of the final oral exam.</td>
</tr>
<tr>
<td><strong>P</strong></td>
<td>Pass on EVERY homework, EVERY presentation and BOTH PARTS of the final oral exam.</td>
</tr>
<tr>
<td><strong>PwD</strong></td>
<td>Requirement for P, plus at least 3 PwD from all the homework and PwD in BOTH PARTS of oral exam.</td>
</tr>
</tbody>
</table>

**Remark 5.6** If a student receives Fx, then a re-exam can be arranged within a week. The
re-exam will be performed and evaluated in the same way as the previous oral exam.

**Remark 5.7** If a student receives at least 5 PwD from all the homework assignments, then
they do not need to solve basic problems in the second part of oral exam as a rewarded bonus.

**Remark 5.8** The evaluation of the final grade is designed in a way that if a student receives
PwD/P in the final grade, then it means that the student must obtain PwD/P level for ILO1-4/ILO1-5. Therefore, the combination of grades from various assessment tasks is consistent
of the grading criteria for ILOs. For example, if a student receives a pass in a homework
assignment, then they must obtain at least 9 points out of 15 points from the basic problems.
To receive 9 points, one can not make too many errors which implies that one has to fulfill
the pass level for ILO3 and ILO4.
6 Reflection

6.1 Motivation of design of criteria

Firstly, since it is a graduate course, I find it unnecessary to evaluate the students by too many levels. Second, ILO1-3 are distinguished by the level of execution since I understand some people might have difficulty with memorizing all the details for the material. It is fine for me if the student makes some minor errors in their answer or requires some hints to respond to the questions that I ask. Also, ILO4 is distinguished by the level of difficulties because there are basic problem and advanced problems involving several parts of the course contents. Lastly, ILO5 is assessed by only one level due to that from my experience, the students always prepare properly before their presentation, so it is more like an opportunity for them to practice their presentation skill than a serious evaluation.

6.2 Feedback from the peers and the student

Feedback from the peers: In both feedback reports from the peers, they pointed out that one of the ILOs from my previous version used non-assessable verb, I agree with that and I have modified it. Another comment from one of the report is that how the point-grading system using in homework assignments and oral exam is linked to the grading criteria for ILOs. I have addressed this issue in more detail in subsection 5.4. One last comment from the reports is that how the students would feel about being evaluated with various levels but in the end only P or F would be assigned in reality. To answer this question, I asked one of my former student, and he agreed that using F/P/PwD system can motivate him to learn more even though PwD is equivalent to P in reality.

Feedback from a former student: The former student thought that the new grading criteria is clearer than before, and it can help the students to understand what they should do exactly in order to pass at the very beginning of the course. With this understanding, every student can know what their learning progress is and can prevent panicking if they could not answer a problem in the oral exam. Moreover, I was thinking about giving 6 basic problems in every homework, but the student indicated that he enjoyed that there were some more challenging problems in his homework when he was taking the course, so I decided to assign 5 basic problems and 2 optional and advanced problems in each homework instead. By doing so, students who are more interested in this course can benefit from the advanced problems and at the same time, I can make sure everyone who passes the course will learn the basics and fulfill all the ILOs. Moreover, a concern from this student was that the formula for combining grades might be a little bit strict since one has to receive a pass in every homework. After some thoughts, I think that the design of 5 basics problems plus 2 advanced problems can also be helpful to address this issue. Apart from this design, I provide a re-submission opportunity for every homework to lower the difficulty of receiving a pass in a homework assignment.

6.3 Grading criteria and students’ expected performance at assessment

At the first meeting of the course, I will explain and go through everything from section 1 to section 5 of this file to the students. In particular, I will make sure they understand section 5, in which I clarified how I evaluate each assessment and how I combine everything into a final grade. For example, I will show them what kind of answer is evaluated by 1, 2 or 3 in a homework problem. Moreover, I will emphasize that memorizing everything is not necessary to pass the course, and I am willing to help or provide some hints if they are stuck, since after
all, all the assessments are designed to help the students to learn more instead of cranking out some numbers.

6.4 Method of combination grades and learning process

I spent some time on thinking about how to combine various assessments. The first way that came to my mind was appointing certain percentage to different assessment tasks, but I found troubles to distribute the percentage in a satisfied way. I would like to emphasize the oral exam, while in the mean time I think homework assignments are also crucial. If I distributed the percentage evenly among the exam and the homework, then how should I further distribute the percentage for the homework among all seven home assignments. That is another troublesome problem to deal with. Luckily, I came up with this new combination method as shown in subsection 5.4. It is not percentage-based but criteria-based combination. One has to pass all the homework, and all the presentations, and the final oral exam to receive a pass in the final grade. This makes every assessment equally important while not diminishing their own individual importance. Doing extremely well in some parts of the assignment tasks is not enough to pass the course since not all ILOs are fulfilled in this case. Thus, what I expect is that this combination method would affect the learning process of a student in the following way: the student has to learn the basics from every topic in order to get a pass on the associated homework. This guarantees that the student fulfills ILO3 and ILO4 for that given topic, and at the same time it avoids the student from leaving behind. Moreover, learning the basics from every topic helps the students to prepare ILO1 and ILO2 as well. Both are assessed in the final oral exam. Lastly, ILO5 are also examined on a regular basis through out the course. Consequently, after the lecture for a certain topic, the combination method can make sure the students satisfy all the ILOs for that given topic.