Third Nordic Logic Summerschool, Stockholm, August 7 – 11, 2017

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Book of Abstracts

Revised version

August 2, 2017

Preface

The Third Nordic Logic Summerschool (NLS 2017) is arranged under the auspices of the Scandinavian Logic Society. The two previous schools were organized in Nordfjordeid, Norway (2013), and in Helsinki (2015) in connection with Logic Colloquium held there. The intended audience is advanced master students, PhD-students, postdocs and experienced researchers wishing to learn the state of the art in a particular subject.

This year's lecture program covers a wide range of topics in mathematical logic, philosophical logic, and computer science logic. The school consists of 10 five hour courses, running in two parallel streams. The lecturers are

– Mirna Džamonja, Professor of Mathematics, University of East Anglia, UK

– Martín Escardó, Reader in Theoretical Computer Science, University of Birmingham, UK

– Henrik Forssell, Researcher, Department of Informatics, Oslo University, Norway

– Volker Halbach, Professor of Philosophy, Oxford University, UK

– Larry Moss, Professor of Mathematics, Indiana University, Bloomington, USA

– Anca Muscholl, Professor, LaBRI, Université Bordeaux, France

– Eric Pacuit, Assistant Professor, Department of Philosophy, University of Maryland, USA

– Peter Pagin, Professor of Theoretical Philosophy, Stockholm University, Sweden

– Sara L. Uckelman, Lecturer, Department of Philosophy, University of Durham, UK

– Dag Westerståhl, Professor of Theoretical Philosophy, Stockholm University, Sweden

– Andreas Weiermann, Professor of Mathematics, University of Ghent, Belgium

The Programme Committee of NLS 2017, responsible for the composition of the program and selection of lecturers, consists of: Thierry Coquand (Göteborg University), Ali Enayat (Göteborg University), Mai Gehrke (IRIF, Paris), Nina Gierasimczuk (Technical University of Denmark), Valentin Goranko (Stockholm University), Lauri Hella (University of Tampere), Lars Kristiansen (Oslo University), Juha Kontinen (Helsinki University), Øystein Linnebo (Oslo University), Sara Negri (Helsinki University), Erik Palmgren (chair, Stockholm University).

In addition, the school has a few sessions for short research presentation intended for participants who wish to present research of their own. The committee that selected these presentations consists of members of the local organization committee for NLS 2017: Valentin Goranko, Dilian Gurov (KTH – Royal Institute of Technology), Roussanka Loukanova (Stockholm University) and Erik Palmgren.

Thanks goes foremost to the lecturers for accepting the program committee's invitation to give these courses, and then to the contributors of short research presentations. Thanks goes to the members of the program committees for their dedicated work. The Logic in Stockholm organization committee is very grateful to the sponsors of NLS 2017: Stockholm Mathematics Centre, Stockholm University, Prover Technology and KTH – Royal Institute of Technology.

Stockholm on June 22, 2017

Valentin Goranko
co-chair of the organization committee of Logic in Stockholm 2017

Erik Palmgren
chair of the program committee for NLS 2017,
co-chair of the organization committee of Logic in Stockholm 2017

Courses

The courses run in two parallel streams, "M" and "P", with regular hours, Monday through Friday, as follows:

9:00 – 10:00	(M)	Mirna Džamonja, *Set Theory: Forcing methods at the successor of a singular cardinal.*
	(P)	Larry Moss, *Natural Logic*
10:20 – 11:20	(M)	Andreas Weiermann, *Proof Theory*
	(P)	Eric Pacuit, *Logic and Probabilistic Models of Belief Change*
11:40 – 12:40	(M)	Henrik Forssell, *Categorical Logic*
	(P)	Sara L. Uckelman, *Introduction to Logic in the Middle Ages*
14:00 – 15:00	(M)	Martín Escardó, *Topological and Constructive Aspects of Higher-Order Computation*
	(P)	Peter Pagin and Dag Westerståhl, *Compositionality*
15:20 – 16:20	(M)	Anca Muscholl, *Logic in Computer Science – Control and Synthesis, from a Distributed Perspective*
	(P)	Volker Halbach, *Truth & Paradox*

Abstracts

Mirna Džamonja, *Set Theory: Forcing methods at the successor of a singular cardinal*

Abstract: Forcing is a method in set theory that changes the properties of the ambient universe, for example the values of the power set function on some set or even a proper class of cardinals. This method is rather well developed when it comes to the successors of regular cardinals, but it is much more challenging to have a method that works at singular cardinals and their successors. In fact, it is known that such a method must necessarily involve the use of large cardinals. In joint work with co-workers James Cummings, Menachem Magidor and Charles Morgan and Saharon Shelah, we have been developing one such method, the details of which will be exposed in the course.

Martín Escardó, *Topological and Constructive Aspects of Higher-Order Computation*

Abstract: In higher-order computability we study computation with infinite objects, such as streams, real numbers, and higher types. Topology plays the role of mediating between the infinite nature of such objects with the finite nature of computers and algorithms. Of particular importance are the Kleene-Kreisel spaces modeling higher types, where topological notions such as continuity and compactness play a significant role in computability considerations. In these lectures we will introduce these and related concepts, their theory and some applications.

Henrik Forssell, *Categorical Logic*

Abstract: Categorical logic studies the interpretation of logical theories in categories and the interplay between formal theories and categories. As such, it is abstract algebraic logic extended to predicate logic and beyond.

Categorical semantics provides a wide variety of models for various formal languages and theories, including for intuitionistic first- and higher-order logic.

Moreover, the categorical framework provides a rich conceptual background for constructions and perspectives that are not (always so easily) obtainable in the more classical model theoretic setting.

This course gives a first introduction to central ideas and concepts in categorical logic. We shall focus primarily on first-order logic, including intuitionistic first-order logic and fragments of first-order logic. Prerequisites will be kept to a minimum, with an initial emphasis on from-the-ground-up explanations of the categorical interpretation of the logical constants, examples of interpretations is various categories, the familiar Tarski and Kripke semantics as special cases, the models-as-functors perspective, and the construction of universal models. On this basis, we will give pointers to and discuss further topics, such as higher-order logic, forcing semantics, duality between syntax and semantics, and type theories (depending on time and interest).

Prerequisites: (Modest) supplementary material will be posted in advance so that it can be assumed that the participants have some familiarity with basic notions of category theory such as category, functor, natural transformation, finite limits and colimits.

Volker Halbach, *Truth & Paradox*

Abstract: The course provides an introduction to the theory of the truth and the semantic paradoxes. The course does not presuppose any knowledge of logic beyond the basics of first-order predicate logic as it is taught in all introductory logic courses. At the beginning basic techniques such as Gödel's diagonal lemma will be proved in a simple theory of syntax without the detour through arithmetization and fundamental results such as Tarski's theorem on the undefinability of truth will be proved in this setting. Yablo's, Curry's, McGee's, and other paradoxes will be analyzed. It will be argued that many of these paradoxes can be reduced to a single source. To this end possible-worlds semantics for truth and other modal predicates will be employed, which allows to visualize the paradoxes in an informative and illuminating way. The machinery that is developed can also shed some light on the notions of self-reference and circularity that are not only central in the theory of paradoxes, but also in the analysis of Gödel's incompleteness theorems and related phenomena. Some popular axiomatic theories of truth will be presented and their main advantages and disadvantages will be discussed. Finally, if time permits, some applications of the theory of truth and the paradoxes, for instance, to the theory of logical consequence will be outlined.

Larry Moss, *Natural Logic*

Abstract: This is a course on 'surface reasoning' in natural language. The overall goal is to study logical systems which are based on natural language rather than (say) first-order logic. Most of the systems are complete and decidable, the class will see a lot of technical work in this direction. At the same time, the work is elementary. One needs to be comfortable with informal proofs, but only a small logic background is technically needed to follow the course.

Specific topics include: extended syllogistic logics; logics including verbs, relative clauses, and relative size quantifiers; the limits of syllogistic logics, monotonicity calculi; algorithms, complexity, and computer implementations.

The course will have a small amount of daily homework to help people learn. It also may involve some work with running computer programs which carry out proof search and model building in natural logics.

The topic of natural logic lends itself to philosophical reflections on the nature of semantics and is arguably something all formally-minded linguists and linguistically-minded logicians should know about. For people coming with a logic background, the course will offer many completeness theorems (as in modal logic), and connections to topics such as: fragments of first-order logic, some connections to combinatorics, and the typed lambda calculus.

Anca Muscholl, *Logic in Computer Science — Control and Synthesis, from a Distributed Perspective*

Abstract: Formal methods in computer science rely on a clear mathematical understanding of programs and their interactions. In synthesis, the goal is to construct a program that complies with some given logical specification. Synthesis of reactive programs addresses this question in the setting where programs interact with their environment. This problem was proposed in the sixties by A. Church as the solvability problem, and it can be also stated in terms of a game between program and environment. An alternative formulation is the theory of supervisory control, that asks to build a controller guaranteeing that a given program satisfies some requirements.

The course will start with an introduction to the areas of logic, automata and game theory that play a prominent role in synthesis. Then we will present some frameworks of distributed synthesis. The goal in distributed synthesis is to construct programs and controllers that consist of local entities that evolve by exchanging information. Distributed synthesis is a truly challenging area, where in some of the frameworks the decidability of the synthesis problem remains open. We will discuss the state-of-the-art and the challenges raised by the distributed setting.

Eric Pacuit, *Logical and Probabilistic Models of Belief Change*

Abstract: Reasoning about the knowledge and beliefs of a single agent or group of agents is an interdisciplinary concern spanning computer science, game theory, philosophy, linguistics and statistics. Inspired, in part, by issues in these different "application" areas, many different notions of knowledge and belief have been identified and analyzed in the formal epistemology literature. The main challenge is not to argue that one particular account of belief or knowledge is primary, but, rather, to explore the logical space of definitions and identify interesting relationships between the different notions. A second challenge (especially for students) is to keep track of the many different formal frameworks used in this broad literature (typical examples include modal logics of knowledge and belief, the theory of subjective probability, but there are many variants, such as the Dempster-Shafer belief functions and conditional probability systems). This foundational course will introduce students to key methodological, conceptual and technical issues that arise when designing a formalism to make precise intuitions about the beliefs of a group of agents, and how these beliefs may change over time. There are two central questions that I will address is this course: 1. What is the precise relationship between the different formalisms describing an agent's beliefs (e.g., what is the relationship between an agent's graded beliefs and full beliefs?); and 2. How should a agent change her beliefs in response to new evidence?

In this course, I will introduce the main formalisms that can describe an agents' beliefs and how those beliefs change over time. Rather than focusing solely on the technical details of a specific formalism, I will pay special attention to the key foundational questions (of course, introducing formal details as needed). There are two goals of this course. The first is to explain the relationship between logical and probabilistic models of belief. The second is to explore the many technical and conceptual issues that arise when studying how agents' beliefs change over time. In addition to introducing the key formal frameworks, this course will introduce the fundamental conceptual questions that drive much of the research in formal epistemology.

Peter Pagin and Dag Westerståhl, *Compositionality*

Abstract: The course gives an introduction to the main idea of the principle of semantic compositionality, some of its formal properties, some variants of the idea, and some interesting applications. The applications come from both logic and natural language semantics. The following classes are planned:

1. The general idea of compositionality. Historical background. Formal definitions. Stronger and weaker versions. Some standard arguments for and against compositionality.

2. Compositionality and context dependence. Extensionality and intensionality. Extra-linguistic context dependence. Linguistic context-dependence and general compositionality.

3. Applications 1: (a) Quotation: non-compositional but general compositional. (b) The problem of compositional accounts of the semantics of belief-sentences. (c) Compositionality solves Carnap's problem: Do the laws of classical propositional logic fix the meaning of the usual connectives?

4. Applications 2: Non-compositionality in logic: IF-logic versus Dependence logic.

5. Compositionality and the computational complexity of interpretation: Is compositional interpretation the most efficient kind of interpretation?

Sara L. Uckelman, *Introduction to Logic in the Middle Ages*

Abstract: In recent years, modern logicians have become increasingly aware of the wealth of developments in logic in the High Middle Ages, the period between roughly 1150 and 1400. Yet, these developments are still inaccessible to those who don't read Latin or Arabic, and even the texts that are translated require an understanding of the Aristotelian background against which the medieval developments occurred. While medieval logic was strongly influenced by Aristotle, the most innovative developments are those which are unAristotelian. The purpose of this course is to introduce the modern logician to the main developments in medieval logic between 1150 and 1400, highlighting aspects such as dynamics and multi-agency that are of particular interest to people working in logic nowadays.

These main developments include:

- Theories of consequences
- Theories of modality
- The *obligationes* disputations
- Analyses of sophisms and paradoxes

No knowledge of Latin or medieval logic is presupposed.

Andreas Weiermann, *Proof Theory*

Abstract: We start with basic material regarding Gentzen's sequent calculus for predicate logic: Cut elimination, Herbrand's theorem, interpolation.

Then we switch to classical results regarding the proof-theoretic analysis of first order Peano aritmetic (PA): provable and unprovable instances of transfinite induction.

In a next step we provide the standard classification of the provably recursive functions of PA.

These results will be put into the phase transition for Gödel incompleteness perspective: Phase transitions for Goodstein sequences and the Paris Harrington theorem.

We will end with a discussion of lower bounds for the lengths of finite proofs in PA of true and purely existential statements.

If time is left over we intend to cover extra material on phase transitions.

Short Research Presentations

Monday Aug 7, stream M (17.40–18.30):

- *A step towards a coordinate free version of Gödel's second theorem*
  Balthasar Grabmayr (Humboldt University of Berlin)
- *Transition operators assigned to physical systems*
  Jan Paseka (Masaryk University) and Ivan Chajda (Palacky University, Olomouc)

Monday Aug 7, stream P (17.40–18.30):

- *Object dependency in Timothy Williamson's deductive argument for necessitism*
  Zuzanna Gnatek (Trinity College Dublin)
- *How to combine entailment and counterfactuals? Relevant logics and the problem of interpreting the sentences of the form a implies b*
  Aleksandra Samonek (Université Catholique de Louvain)

Tuesday Aug 8 (17.40 – 18.30, 10 minute poster sessions):

- *Reasoning about belief based on information fusion*
  Tuan-Fang Fan (National Penghu University of Science and Technology) and Churn-Junh Liau (Academia Sinica)
- *Abductive reasoning with description logics*
  Julia Pukancova (Comenius University in Bratislava)
- *Unpacking Broome's philosophical account of reasoning: a formal framework*
  Antonis Staras (University of East Anglia)
- *Analysing ranking-based semantics in logic-based argumentation with existential rules*
  Bruno Yun (University of Montpellier)

Thursday Aug 10, stream M (17.40–18.40):

- *Alpha-recursion and Randomness*
  Paul-Elliot Anglès d'Auriac (LACL, France)

- *Infinite time Turing machines: what about gaps in the clockable ordinals?*
  Sabrina Ouazzani (LACL, France)

Thursday Aug 10, stream P (17.40–18.40):

- *Oppositions Within and Among Logics.*
  Carolina Blasio (Unicamp, Brazil)

- *Models of Disquotational Truth.*
  Michał Tomasz Godziszewski

- *Games for Minimal Logic.*
  Alexandra Pavlova (Université Paris 1 Panthéon-Sorbonne)

Abstracts of Research Presentations

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Paul Elliot Anglès d'Auriac
(LACL,France)

Randomness in α-recursion

July 27, 2017

Abstract

Algorithmic randomness is the field that studies random reals in the recursion theoretic point of view. It defines a random real as one that has no exceptionnal but sufficiently simple property. By considering metarecursion, a notion derived from descriptive set theory, higher notions of randomness as Π^1_1-randomness have been defined. However, there is an even more general notion of computation, namely α-recursion, that includes metarecursion and Infinite Time Turing Machine computation. This gives a framework for defining new randomness notions.

In this talk, I will first define α-recursion and give some examples for particular α, then, I will show how to use it to define randomness notions.

Oppositions within and Among Logics

Carolina Blasio
Unicamp, Brazil

April 24, 2017

Abstract. In this talk I propose a formal definition of oppositions between logical statements, between connectives, and between logics. I also present a procedure for producing the dual, the antonym and the contradictory of statements *within* a given logic, as well as oppositions *among* logics. Examples of opposition within a given logic are the duality between the law of excluded middle and the principle of explosion in classic logic, and the duality between the conjunction and the disjunction. For oppositions among logics one can state the characteristic opposition between paraconsistent and paracomplete logics. To create a procedure for producing oppositions, I extend the abstract formal definition of duality proposed by [1], who explored the symmetry of the multiple-conclusion consequence relation, to a two-dimensional notion of consequence relation, the B-consequence, defined in [2]. In addition to the duality I will explore other oppositions relations of the Square of Oppositions. I notice that, in the case of non-classical logics, the procedure shows that what are usually called "dual logics" in the literature consist of a pool of different kinds of opposition relations.

References:

[1] Joao Marcos. Ineffable inconsistencies. In: J.-Y. Beziau, W A Carnielli, and D Gabbay, editors, **Handbook of Paraconsistency,** volume 9 of Studies in Logic, pages 301–311. North Holland, Amsterdam, 2007.

[2] Carolina Blasio, Joao Marcos and Heinrich Wansing. **An inferentially many-valued two-dimensional notion of entailment.** Unpublished.

Transition operators assigned to physical systems

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Keywords: Physical system, transition relation, transition operators, states, complete lattice, transition frame, logic of quantum mechanics.

In 1900, D. Hilbert formulated his famous 23 problems. In the problem number 6, he asked: "Can physics be axiomatized?" It means that he asked if physics can be formalized and/or axiomatized for to reach a logically perfect system forming a basis of precise physical reasoning. This challenge was followed by G. Birkhoff and J. von Neuman in 1930s producing the so-called logic of quantum mechanics (see [1]). We are going to addopt a method and examples of D. J. Foulis, however, we are not restricted to the logic of quantum mechanics. We are focused on a general situation with a physical system endowed with states which it can reach. Our goal is to assign to every such a system the so-called transition operators completely determining its transition relation (see [2]). Conditions under which this assignment works perfectly will be formulated.

We start with a formalization of a given physical system. Every physical system is described by certain quantities and states through them it goes. From the logical point of view, we can formulate propositions saying what a quantity in a given state is.

Denote by S the set of states of a given physical system $\mathcal{P}$. It is given by the nature of $\mathcal{P}$ from what state $s \in S$ the system $\mathcal{P}$ can go to a state $t \in S$. Hence, there exists a binary relation R on S such that $(s,t) \in R$. This process is called a *transition* of $\mathcal{P}$.

In addition to the previous, the observer of $\mathcal{P}$ can formulate propositions revealing our knowledge about the system. The truth-values of these propositions depend on states. For example, the proposition p can be true if the system $\mathcal{P}$ is in the state s_1 but false if $\mathcal{P}$ is in the state s_2. Hence, for each state $s \in S$ we can evaluate the truth-value of p, it is denoted by $p(s)$. The set of all truth-values for all propositions will be called the *table*. Denote by B the set of propositions about the physical system $\mathcal{P}$ formulated by the observer.

We can introduce a partial order $\leq$ on B as follows:

$$\text{for } p,q \in B, p \leq q \text{ if and only if } p(s) \leq q(s) \text{ for all } s \in S.$$

One can immediately check that the *contradiction*, i.e., the proposition with constant truth value 0, is the least element and the *tautology*, i.e., the proposition with the constant truth value 1 is the greatest element of the partially ordered set $(B;\leq)$; this fact will be expressed by the notation $\mathbf{B} = (B;\leq,0,1)$ for the bounded partially ordered set of propositions about $\mathcal{P}$. This partially ordered set $\mathbf{B} = (B;\leq,0,1)$ will be referred to as a *logic of* $\mathcal{P}$.

However, our physical system $\mathcal{P}$ is dynamical which is captured by its transition and it is described by the transition relation R. Our aim is to set up a dynamic logic based on $(B;\leq,0,1)$ which can formalize the process of transition.

Assume that we are given a partially ordered set $\mathbf{M} = (M;\leq,0,1)$ which will play the role of the set of truth values of our logic. Then we will assign to $(B;\leq,0,1)$ an operator $T\colon B \to M^S$. This means that T transforms every proposition from B into a sequence of truth values (into a sequence of 0's and 1's if our logic is two-valued).

This sequence can be considered again as a proposition on $\mathcal{P}$ depending on states from S but it need not belong to B in general. Such an operator will be called a *transition operator* of $(B; \leq, 0, 1)$ if it is in accordance with the transition relation R (see [2]). Then the formal system $(B; \leq, 0, 1, T)$ will be considered as a *dynamic logic* describing the behaviour of our physical system $\mathcal{P}$.

Specifically, assume that $M = \{0, 1\}$, i.e., our logic is two-valued and $\{0, 1\}^S$ can be identified as usual with the Boolean algebra of all subsets of S. Now, we will describe the role of our transition operator $T \colon B \to \{0, 1\}^S$. Let $s \in S$ be a state and $p \in B$ a proposition from our logic $\mathbf{B}$ such that $T(p)$ is true in the state s, i.e., $s \in T(p)$ when $T(p)$ is understood as a subset of S. If we proceed with a transition from s to t, we have that p will be true in the state t, i.e., $t \in p$ when p is understood as a subset of S.

It is worth noting that the transition operator T is just logical, i.e., a formal operator on the set of propositions B and it need not have a physical interpretation within our logic $\mathbf{B}$.

Our main goal is to determine conditions on the logic $(B; \leq, 0, 1)$ such that the transition operator T will capture the whole information on the transition relation R, which means that the relation R can be recovered by the operator T.

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References

[1] Birkhoff, G. and von Neumann, J. (1936), The Logic of Quantum Mechanics. *Annals of Mathematics* **37**, 823–843.

[2] Chajda, I. and Paseka, J. (2016), Transition operators assigned to physical systems. *Reports on Mathematical Physics*, **78**, 259–280.

Reasoning about Belief based on Information Fusion

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In past decades, data-driven approach has played a pivotal role in the research of AI. In intelligent systems, an agent always makes decision based on information received from different data sources. For such applications, data serves as evidential supports for the agent's decision. In other words, an agent forms her beliefs by pooling together different pieces of evidence from multiple data sources. Hence, it is crucially important for an agent to reason about her belief based on information fusion. Since the seminal work by Hintikka [7], modal logic has been a standard formalism for reasoning about knowledge and belief and its applications to AI and computer science have been extensively explored [4, 8]. However, modal logic suffers from the notorious logical omniscience problem which means that an agent knowing a fact can know all logical consequences of the fact. While this is reasonable for ideal agents, it is very impractical for resource-bounded agents. Hence, modal logic is actually a formalism for representing implicit belief in which modal formula $B\varphi$ is interpreted as "φ is believable". Consequently, in such formalism, we cannot keep track of evidential information that supports an agent's belief.

While modal logic lacks the necessary mechanism for representing the supporting reasons of belief, justification logics (JL) supply the missing component by adding justification terms to modal formulas [3, 1, 2, 6]. However, because JL does not make a distinction between potential and actual evidence, it cannot represent different pieces of information that support an agent's belief with different degrees.

In this paper, we propose an extension of modal logic and JL to address the reason maintenance issue of belief reasoning. The main idea is to enrich JL with modalities for expressing the informational contents of evidence [5] and a predicate for asserting the actual observation of evidence. A Hilbert-style axiomatic system is presented in which the main axiom assures that once a piece of evidence is observed, its informational contents are assimilated into the agent's belief. Then, we extend the basic logic to accommodate possibilistic uncertainty of data. In the extended formalism, we can model the strength of an agent's belief based on different degrees of evidential supports.

More specifically, to represent and reason with an agent's belief based on evidence, we propose a reason-maintenance belief logic (RBL). In the language of RBL, the formation rules of justification terms and formulas are defined as follows:

$$t ::= a \mid x \mid t \cdot t \mid t + t,$$

$$\varphi ::= p \mid \bot \mid \varphi \to \varphi \mid t{:}\varphi \mid B\varphi \mid I_t\varphi \mid \mathrm{O}(t),$$

where $p \in \Phi$, a is a justification constant, and x is a justification variable. We use Tm to denote the set of all justification terms. The resultant language is denoted by $\mathcal{L}_{rb}$. In the logical language, the justification formula $t:\varphi$ is reserved for representing the admissibility of t with respect to φ. That is, $t:\varphi$ means that t is a good reason for believing φ. However, unlike in JL, $t:\varphi$ does not imply that the agent believes φ automatically. The agent will believe it only when the evidence t has been observed, i.e., $\mathrm{o}(t)$ holds. In the logic, each justification term t and the corresponding I_t represent a piece of evidence and its informational contents respectively.

For the semantics of RBL, its model is defined as a tuple

$$\mathfrak{M} = \langle W, R, (R_t)_{t\in\mathsf{Tm}}, E, O, \Vdash\rangle,$$

where W, R, E, and $\Vdash$ are the same as in a Fitting model of JL, $R_t \subseteq W\times W$ is a binary relation for each justification term t such that the coherence condition $R_{s+t} = R_{s\cdot t} = R_s \cap R_t$ is satisfied, and $O: W \to \mathsf{Tm}$ is a function such that for each $w \in W$, $O(w)$ is closed under $\cdot$ and $+$. Intuitively, $O(w)$ means the set of evidence that have been directly observed in w. The closure condition means that, if both pieces of evidence s and t are actually observed in w, then the pieces of compound evidence $s+t$ and $s\cdot t$ are also regarded as being actually observed. In addition, we require that a RBL model must satisfy that $R(w) \subseteq \bigcap_{t\in O(w)} R_t(w)$ for each $w \in W$. Therefore, if a piece of evidence has been directly observed in w, then its informational contents should be assimilated into the agent's belief.

The forcing relation $\Vdash$ is extended to a binary relation between W and $\mathcal{L}_{rb}$ as follows:

- $w \Vdash t:\varphi$ iff $w \in E(t,\varphi)$,
- $w \Vdash B\varphi$ iff for any u such that $(w,u) \in R$, $u \Vdash \varphi$,
- $w \Vdash I_t\varphi$ iff for any u such that $(w,u) \in R_t$, $u \Vdash \varphi$,
- $w \Vdash \mathrm{o}(t)$ iff $t \in O(w)$

We present an axiomatic system and shows several properties of the logic. Then, we also extend the basic formalism to a reason-maintenance possibilistic belief logic in which we can represent and reason about uncertain evidence and belief.

References

1. Artemov, S.: The logic of justification. The Review of Symbolic Logic 1, 477–513 (2008)
2. Artemov, S., Fitting, M.: Justification logic. In: Zalta, E. (ed.) The Stanford Encyclopedia of Philosophy. Fall 2012 edn. (2012)
3. Artemov, S., Nogina, E.: Introducing justification into epistemic logic. Journal of Logic and Computation 15(6), 1059–1073 (2005)
4. Fagin, R., Halpern, J., Moses, Y., Vardi, M.: Reasoning about Knowledge. MIT Press (1996)
5. Fan, T., Liau, C.: Reasoning about justified belief based on the fusion of evidence. In: Proceedings of the 15th European Conference on Logics in Artificial Intelligence (JELIA). pp. 240–255. Lecture Notes in Artificial Intelligence, Springer (2016)
6. Fitting, M.: The logic of proofs, semantically. Annals of Pure and Applied Logic 132(1), 1–25 (2005)
7. Hintikka, J.: Knowledge and Belief. Cornell University Press (1962)
8. Meyer, J.J.C., van der Hoek, W.: Epistemic Logic for AI and Computer Science. Cambridge University Press (1995)

Zuzanna Gnatek (Trinity College Dublin, Ireland)

Object Dependency in Timothy Williamson's Deductive Argument for Necessitism

In his 'Modal Logic as Metaphysics' (2013) Timothy Williamson famously argues for necessitism, that is, for a view according to which necessarily, everything is necessarily something. His argument is mostly abductive, that is, it appeals to the cost that logic and metalogic have to bear if the thesis of necessitism is rejected. But he also presents a straightforward, deductive argument.

One of instances of this argument refers to quantification into singular noun phrase ("the proposition that…"). The central thesis of necessitism

NNE $\Box \forall y \Box \exists x\, x=y$

follows here from the following three premises:

P1 $\Box \forall y \Box (\sim\exists x\, x=y \rightarrow T\, \pi(\sim\exists x\, x=y))$

P2 $\Box \forall y \Box (T\, \pi(\sim\exists x\, x=y) \rightarrow \exists x\, x= \pi (\sim\exists x\, x=y))$

P3 $\Box \forall y \Box (\exists x\, x= \pi (\sim\exists x\, x=y) \rightarrow \exists x\, x=y)$

where an operator *π* applies to a formula *A* to give a singular term *π(A)* denoting the proposition that *A* expresses - "the proposition that *A*", and *T* is a truth predicate.

By P1, if you were nothing, the proposition that you were nothing would be true. By P2, if the proposition that you were nothing were true, that proposition would be something. By P3, if the proposition that you were nothing were something, you would be something.

If we instantiate a variable with a proper name, such as "Socrates", in this argument, we arrive to the famous argument that concludes that necessarily, Socrates is something:

(1) Necessarily, if Socrates is nothing then the proposition that Socrates is nothing is true.
(2) Necessarily, if the proposition that Socrates is nothing is true then the proposition that Socrates is nothing is something.
(3) Necessarily, if the proposition that Socrates is nothing is something then Socrates is something.

Therefore,

(4) Necessarily, Socrates is something.

Williamson shows that premises (1), (2), and (3) together yield (4) - by standard quantified modal logic. Nevertheless, this argument seems to encounter some difficulties. In my talk I would like to discuss them - focusing mostly on the problems raised by the third premise of Williamson's proof, which states that necessarily, if the proposition that Socrates is nothing is something then Socrates is something, and thus presupposes the Object Dependency Principle (OD) for propositions. (According to which, roughly, if the proposition that *P(o)* exists then *o* exists, where *o* is to be replaced by a singular term, and *P(o)* - by a sentence which has that singular term as a constituent.)

It may be argued that OD makes Williamson's proof either inconsistent or circular. In order to explain why this is so in a detailed way I shall present a recast version of the third premise of the proof and two ways of interpreting it that are due to two different notions of the truth-operator involved in this recast version (it might be understood as either redundant or non-redundant). Such a strategy draws on Rumfitt's criticism of an earlier version of Williamson's proof and it reveals some more general problems, such as that there is some equivocation involved in the proof or that the proof cannot hold for metaphysical necessity.

I also consider two responses that a necessitist might provide to defend Williamson's proof against such a strategy (one of them appeals to a possible different way of interpreting the truth-operator which would not lead to such difficulties; another one suggests that the truth-operator need not be involved in the proof at all) together with some difficulties that they encounter.

Selected references

Rumfitt, I., 2003. Contingent Existents. *Philosophy* 78:306, pp. 461-481.

Williamson, T., 2002. Necessary Existents. In *Logic, Thought and Language*, ed. by O'Hear, A., Cambridge University Press, pp. 233-252.

Williamson, T., 2010. Objects, Properties and Contingent Existence. Unpublished.

Williamson, T., 2013. Modal Logic as Metaphysics, Oxford University Press.

Models of Disquotational Truth

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April 25, 2017

Abstract. An axiomatic theory of truth is a formal deductive theory where the property of a sentence being true is treated as a primitive undefined predicate. Logical properties of many axiom systems for the truth predicate have been discussed in the context of the so-called truth-theoretic deflationism, i.e. a view according to which truth is a 'thin' or 'innocent' property without any explanatory or justificary power with respect to non-semantic facts. I will discuss the state of the art concerning proof-theoretic and model-theoretic properties of the so-called disquotational theories of truth, focusing in particular on the problem of syntactic and semantic conservativeness of truth theories over a base (arithmetical) theory (treated mostly as a theory of syntax). I will relate the results to the research on satisfaction classes in models of arithmetic.

I intend to present: (1) a new, easier proof of a recent result by Łełyk and Wcisło: that there exist models of local disquotational theory of arithmetical truth (TB) that are not recursively saturated, (2) a strengthening of result by Cieśliński and Engstrom: that not only PA and TA (True Arithmetic), but actually every complete consistent extension of PA has a model that does not expand to a model of TB.

Balthasar Grabmayr

A Step Towards a Coordinate Free Version of Gödel's Second Theorem

A prevalent interpretation of Gödel's Second Theorem is that a sufficiently adequate and consistent theory T does not prove its own consistency. In this talk, the justification of this philosophical interpretation will be examined. Detlefsen's Stability Problem (Detlefsen, 1986) challenges such a justification by requiring that *every* formula (in the language of T) expressing T-consistency has to be shown to be unprovable in T. The stability problem will be considered by focusing on Gödel numberings, which can be seen as arbitrarily chosen "coordinate systems" in the process of arithmetisation. It will be argued that a satisfactory solution of the stability problem has to be based on a version of Gödel's Second Theorem which does not depend on the choice of such a coordinate system. A solution of this kind will be proposed by proving the invariance of Gödel's Second Theorem with regard to acceptable numberings. This involves two steps. Firstly, the notion of acceptability of a numbering will be discussed. It will be argued that the computability of a numbering is a necessary condition for its acceptability. A precise notion of computability will then allow formerly vague invariance claims to be restated as (meta-)mathematical theorems, whose proofs will be outlined in the second part of the talk. Time permitting, the talk will be concluded by a discussion of the intensionality of the employed consistency statements with reference to work by Halbach and Visser (2014).

At first glance, any injective function from a set of expressions to $\mathbb{N}$ qualifies as a numbering. However, it is not difficult to construct a deviant numbering of the set of arithmetical expressions which allows a binumeration of the provability predicate (satisfying Löb's conditions), thus contradicting Gödel's Second Theorem. In order to avoid trivialising the problem of invariance, certain adequacy conditions for *acceptable* numberings will be presented. It is sufficient for the purposes of this talk to require the following condition: *every acceptable numbering is computable*.

Since a numbering assigns natural numbers to expressions, the explication of the notion *computable* for such functions by means of the Church-Turing Thesis is not straightforward. However, Boker and Dershowitz (2008) provide a suitable framework, basing the concept of computability on finite constructibility. In order to suit this framework, the expressions are taken here to be constructed from a finite "protoalphabet", following Quine (1940).

Since the aim of the present work is to eliminate arbitrary choices in the process of arithmetisation, sets of expressions will be construed as free algebras over such a protoalphabet. This provides a unified account of Gödel numberings, independent of the specific structure of expressions, i.e. whether expressions are finite sequences, trees, sets, etc. It may be further noted, that this approach does not require acceptable numberings to be monotone.

Let $\mathcal{E}$ be any set of expressions (as specified above). It can then be shown that for any two acceptable numberings α and β of $\mathcal{E}$, both $\alpha \circ \beta^{-1}$ and $\beta \circ \alpha^{-1}$ are recursive functions. In this case, α and β are called (computably) equivalent.

Using basic recursion theoretic properties, this result yields for instance the invariance of Tarski's Theorem with regard to acceptable numberings. In order to prove the invariance of Gödel's Second Theorem, certain properties of equivalence of numberings will be shown to be derivable in EA.[1] Let $\ulcorner \phi \urcorner^\alpha$ denote $\overline{\alpha(\phi)}$, i.e. the standard numeral of the α-code of ϕ.

[1] $\mathsf{EA} = I\Delta_0 + \forall x, y \exists! z\, e(x, y, z)$, where $e(x, y, z)$ is a binumeration of the exponentiation function and $I\Delta_0$ is PA with induction restricted to Δ_0-formulæ.

Theorem 1 *For all acceptable numberings α, consistent, recursively enumerable theories $T \supseteq \mathsf{EA}$ and arithmetical formulæ $\mathsf{Pr}_T^\alpha(x)$ satisfying Löb's derivability conditions relative to α (for T), it holds that $T \nvdash \neg\mathsf{Pr}_T^\alpha(\ulcorner 0 = 1 \urcorner^\alpha)$.*

It could be argued that only the usually employed, standard numberings yield predicates expressing provability. However, the next theorem will show that each acceptable numbering α allows the construction of a (non-trivial[2]) provability predicate satisfying Löb's derivability conditions relative to α. The above coordinate free version of Gödel's Second Theorem can thus be seen to properly extend classical versions based on a specific numbering.

Theorem 2 *For all acceptable numberings α and consistent, recursively enumerable theories $T \supseteq \mathsf{EA}$, there exists a formula $\mathsf{Pr}_T^\alpha(x)$ which satisfies Löb's derivability conditions relative to α (for T) and numerates $\{\alpha(\phi) \mid T \vdash \phi\}$ in EA.*

The present work may be viewed as in line with other attempts to eliminate arbitrary choices in the process of arithmetisation. Visser (2011) locates three sources of indeterminacy in the formalization of a consistency statement for a theory T: (I) the choice of a proof system, (II) the choice of a coding system and (III) the choice of a specific formula representing the axiom set of T.

According to Visser (2011), "Feferman's solution (Feferman, 1960) to deal with the indeterminacy is to employ a fixed choice for (I) and (II) and to make (III) part of the individuation of the theory" (p. 544). Visser's (2011) own approach rests on fixed choices for (II) and (III) but is independent of (I). The primary result of the present work is to eliminate the dependency on (II).

Preferred format of presentation: 20 minutes talk.

References

Boker, U. and Dershowitz, N. (2008). The church-turing thesis over arbitrary domains. In Avron, A., Dershowitz, N., and Rabinovich, A., editors, *Pillars of Computer Science*, pages 199–229. Springer-Verlag, Berlin, Heidelberg.

Detlefsen, M. (1986). *Hilbert's Program - An Essay on Mathematical Instrumentalism*. Springer Science and Business Media, Berlin Heidelberg.

Feferman, S. (1960). Arithmetization of metamathematics in a general setting. *Fundamenta Mathematicae*, 49:35–92.

Halbach, V. and Visser, A. (2014). Self-reference in arithmetic i. *Review of Symbolic Logic*, 7(4):671–691.

Quine, W. V. (1940). *Mathematical Logic*. W. W. Norton, New York.

Visser, A. (2011). Can we make the second incompleteness theorem coordinate free? *Journal of Logic and Computation*, 21(4):543–560.

[2] Note that also the formula $x = x$ satisfies Löb's derivability conditions.

Infinite time Turing machines: what about gaps in the clockable ordinals?

Sabrina Ouazzani

LACL, Paris-Est Créteil University

I will present you a part of my PhD thesis entitled "From algorithmics to logics through infinite time computation" in which I have studied the infinite time Turing machines model of computation from a computer scientist point of view. In particular I focused on the structure of gaps in the clockable ordinals, that is to say, ordinal times at which no infinite time program halts.

Infinite time Turing machines are a generalisation of classical Turing machines to ordinal times of computation, introduced in 2000 by Hamkins and Lewis in [HL00]. Infinite time Turing machines have right infinite tapes on a binary alphabet $\{0, 1\}$ (thus each tape contains a "real"), a common head and a finite number of states. A special limit operation is defined to settle the hardware of the machine at limit steps of computation (head rewound to the origin cell, in a lim state, each cell of each tape changed to the lim sup value of its whole history). Halting is defined by entering a transition leading to the halting state. Clockable ordinals are the ordinals corresponding to halting times of infinite time programs on empty input. Gaps are intervals of non-clockable ordinals.

In this talk I will present infinite time Turing machines (ITTM), from the original definition of the model to some new infinite time algorithms. I will show you some information about the structure of gaps and the properties of the ITTM-computable ordinals, most of them obtained by such algorithmic techniques. Main results presented here can be found in [Wel09], [HL00] and [CDLO17].

References

[CDLO17] Merlin Carl, Bruno Durand, Grégory Lafitte, and Sabrina Ouazzani. Admissibles in gaps. In *Unveiling Dynamics and Complexity - 13th Conference on Computability in Europe, CIE 2017, Turku, Finland, June 12-16, 2017*, 2017.

[HL00] Joel D. Hamkins and Andrew Lewis. Infinite time turing machines. *Journal of Symbolic Logic*, 65(2):567–604, 2000.

[Wel09] Philip D. Welch. Characteristics of discrete transfinite time turing machine models: Halting times, stabilization times, and normal form theorems. *Theoretical Computer Science*, 410(4-5):426–442, 2009.

Alexandra Pavlova

Dialogue Games for Minimal Logic

Introduction

By dialogue games we understand dialogue logic of P. Lorenzen and K. Lorenz which defines validity. Several authors proposed their proofs for the intuitionistic dialogues, such as Fermüller [3], Felscher [2], Sørensen and Urzyczyn [10], [1].

We define a class of dialogue games for minimal logic and a corresponding sequent calculus. We define a sequent calculus for minimal logic as an intuitionist calculus without *the right weakening* (WR) of the form:

$$\frac{\Gamma \longrightarrow \varnothing}{\Gamma \longrightarrow \mathfrak{D}}\ WR\emptyset$$

where $\mathfrak{D}$ is an arbitrary formula. Finally, we establish the correspondence between the winning strategies for the Proponent in that class of games and the validity in minimal propositional logic.

1 Sequent Calculus for Minimal Logic $G_3^{min}a$

Our calculus is based on the Klenee logic G_3a. We restrict the usage of the *right weakening* structural rule (WR). There are no structural rules, but we keep the main formula in the premisses.

Definition 1 *The axiom of the system $G_3^{min}a$ is*

$$\mathfrak{A}, \Gamma \longrightarrow \mathfrak{A} \qquad (Ax.^{Int})$$

Definition 2 *The system $G_3^{min}a$ has the following rules of inference:*

$$\frac{\mathfrak{A}, \Gamma \longrightarrow \mathfrak{B}}{\Gamma \longrightarrow \mathfrak{A} \supset \mathfrak{B}}\ \supset^{S+} \qquad \frac{\mathfrak{A} \supset \mathfrak{B}, \Gamma \longrightarrow \mathfrak{A} \quad and \quad \mathfrak{B}, \mathfrak{A} \supset \mathfrak{B}, \Gamma \longrightarrow \Theta}{\mathfrak{A} \supset \mathfrak{B}, \Gamma \longrightarrow \Theta}\ \supset^{A+}$$

$$\frac{\Gamma \longrightarrow \mathfrak{A} \quad and \quad \Gamma \longrightarrow \mathfrak{B}}{\Gamma \longrightarrow \mathfrak{A} \wedge \mathfrak{B}}\ \wedge^{S+} \qquad \frac{\mathfrak{A}, \mathfrak{A} \wedge \mathfrak{B}, \Gamma \longrightarrow \Theta \quad or \quad \mathfrak{B}, \mathfrak{A} \wedge \mathfrak{B}, \Gamma \longrightarrow \Theta}{\mathfrak{A} \wedge \mathfrak{B}, \Gamma \longrightarrow \Theta}\ \wedge^{A+}$$

$$\frac{\Gamma \longrightarrow \mathfrak{A} \quad or \quad \Gamma \longrightarrow \mathfrak{B}}{\Gamma \longrightarrow \mathfrak{A} \vee \mathfrak{B}}\ \vee^{S+} \qquad \frac{\mathfrak{A}, \mathfrak{A} \vee \mathfrak{B}, \Gamma \longrightarrow \Theta \quad and \quad \mathfrak{B}, \mathfrak{A} \vee \mathfrak{B}, \Gamma \longrightarrow \Theta}{\mathfrak{A} \vee \mathfrak{B}, \Gamma \longrightarrow \Theta}\ \vee^{A+}$$

$$\frac{\mathfrak{A}, \Gamma \longrightarrow}{\Gamma \longrightarrow \neg\mathfrak{A}}\ \neg^{S+} \qquad \frac{\neg\mathfrak{A}, \Gamma \longrightarrow \mathfrak{A}}{\neg\mathfrak{A}, \Gamma \longrightarrow \Theta}\ \neg^{A+}$$

where Θ is empty ($\Theta = \emptyset$).

2 Minimal Dialogue Logic D^{min}

We base our system on the Intuitionistic Dialogue Logic as it is presented by Krabbe in [6]. Dialogue is a two-player game about some formula with the *Proponent* (P) and the *Opponent* (O). Normally, there are two following levels of rules in dialogue logic:

Definition 3 (Logical Rules) *The system $D^{min}a$ has the following logical rules:*

Connective	***Attack***	***Defence***
$X - ! - \mathfrak{A} \wedge \mathfrak{B}$	$Y - ? - \wedge_L$	$X - ! - \mathfrak{A}$
	$Y - ? - \wedge_R$	$X - ! - \mathfrak{B}$
$X - ! - \mathfrak{A} \vee \mathfrak{B}$	$Y - ? - \vee$	$X - ! - \mathfrak{A}$
		$X - ! - \mathfrak{B}$
$X - ! - \mathfrak{A} \supset \mathfrak{B}$	$Y - ! - \mathfrak{A}$	$X - ! - \mathfrak{B}$
$X - ! - \neg\mathfrak{A}$	$Y - ! - \mathfrak{A}$	-
$X - ! - \forall\mathfrak{x}\mathfrak{A}(\mathfrak{x})$	$Y - ? - \forall\mathfrak{x}/\mathfrak{n}$	$X - ! - \mathfrak{A}[\mathfrak{n}/\mathfrak{x}]$
$X - ! - \exists\mathfrak{x}\mathfrak{A}(\mathfrak{x})$	$Y - ? - \exists\mathfrak{x}$	$X - ! - \mathfrak{A}[\mathfrak{n}/\mathfrak{x}]$

Definition 4 *A dialogue is a sequence of attacks and defences that begins with a finite (possibly empty) multiset Π of formulae that are initially granted by O and a finite (nonempty) multiset Δ of formulae that are initially disputed by O.*

Definition 5 (Structural rules) *The system $D^{min}a$ has the following structural rules:*

Start *The first move of the dialogue is carried out by O and consists in an attack on (the unique) initially disputed formula $\mathfrak{A}$[1].*

[1] We count as a zero step the one where P proposes a formula for the dispute.

Alternation *Moves strictly alternate between players O and P.*

Atom *Atomic formulas, including $\bot$, may be stated only by O.*

D11 *If it is X's turn and there are more than one attacks by Y that X has not yet defended, only the most recent one may be defended*[2].

D12 *Any attack may be defended at most once*[3].

Attack-rule *O can attack P's one and the same formula only once, whereas P can attack O's formula several times.*

Minimal rule *Each attack should be defended if it is possible according to the logical rules.*

Definition 6 (Ending) *The game ends if and only if the player whose turn it is to move has no legal move to make.*

Definition 7 (Winning conditions) *The game ends with P winning iff it is O's turn and she has no possible move left to make.*

The game ends with O winning iff it is P's turn and she has no possible move left to make.

A round consists of an attack of X and a defence of Y, or just an attack.

3 The Correspondence between $G_3^{min}a$ and D^{min}

Definition 8 *A dialogue tree T for a dialogue sequent $\Pi \longrightarrow \mathfrak{A}$ is a rooted directed tree whose nodes are rounds in a dialogue game such that every branch of T is a dialogue with initially granted formulas Π and initially disputed formula $\mathfrak{A}$.*

Definition 9 (Winning Strategy) *A finite dialogue tree T is a winning strategy τ for X if and only if each branch that is the result of X's choice ends with the move of X, i.e. player Y has no possible move to make.*

Theorem 1 (Minimal validity). *Let A be any formula of propositional logic. The following conditions are equivalent:*

1. *There is a winning strategy for Proponent in dialogue $\mathcal{D}(A)$;*
2. *There exists a $G_3^{min}a$ derivation of the formula A (i.e., $\Gamma \longrightarrow A$, where γ is empty). Furtheremore, there exists an algorithm turning Proponent's winning strategy into the $G_3^{min}a$ derivation and visa versa.*

However, any derivation in $G_3^{min}a$ can be transformed into the minimal dialogue logic D^{min} and visa versa.

Theorem 2 (Correspondence result). *Every winning strategy τ for $\mathcal{D}(A, \Gamma)$ (i.e., for a dialogue with initially disputed formula A, where the Opponent initially grants the formulae in the multiset Γ) can be transformed into a $G_3^{min}a$ derivation of $\Gamma \longrightarrow A$ and visa versa.*

References

1. Alama J., Knoks A., Uckelman S.L. (2011): "Dialogue Games for Classical Logic" (short paper). In M. Giese and R. Kuznets (Eds.), *TABLEAUX 2011: Workshops, Tutorials, and Short Papers* (pp. 82–86).
2. Felscher W. (1985): "Dialogues, strategies, and intuitionistic provability". *Annals of Pure and Applied Logic 28*, pp. 217–254.
3. Fermüller C.G. (2003): "Parallel Dialogue Games and Hypersequents for Intermediate Logics". In M.C. Mayer, F. Pirri (Eds.), *TABLEAUX 2003 Automated Reasoning with Analytic Tableaux and Related Methods* (pp. 48–64).
4. Gentzen Gerhard Karl Erich (1934): "Untersuchungen über das logische Schließen. I, II". *Mathematische Zeitschrift.* 39 (2): 176–210.
5. Kleene S. C. (1952): *Introduction to Metamathematics*, the Netherlands.
6. Krabbe E.C.W. (2006): "Dialogue Logic". In D.M. Gabbay and J. Woods (Eds.), *Handbook of the History of Logic* (Vol. 7, pp. 665–704). New York: Elsevier.
7. Lorenzen P. and Lorenz K. (1978): *Dialogische Logik*. Wissenschaftlische Buchgesellschaft, Darmstadt
8. Pavlova A.M. (2015): "Truth in Dialogue Logic and Game-Theoretical Semantics (GTS)", in *Logical Investigations.* vol. 21, no 2, pp. 107–133.
9. Rahman S., Tulenheimo T. (2009): "From Games to Dialogues and Back: Towards a General Frame for Validity". In *Games: Unifying logic, Lan- guage and Philosophy.* Springer.
10. Sørensen M.H., Urzyczyn P. (2007): "Sequent calculus, dialogues, and cut elimination". *In: Reflections on Type Theory, λ-Calculus, and the Mind*, pp. 253–261. Universiteit Nijmegen.
11. Troelstra A.S., Schwichtenberg H.(2000): *Basic Proof Theory.* Cambridge University Press, 2nd edn.

[2] This is a rule for minimal and intuitionistic logic only. In classical logic any attack can be defended.

[3] This rule is applicable only for minimal and intuitionistic calculus, but in classical one P can repeat his defences.

Abductive Reasoning with Description Logics
(Extended Abstract)

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1 Introduction

Abduction, i.e., the problem of explaining what is missing in a knowledge base $\mathcal{K}$, from which some observation O does not follow (Peirce, 1878), only recently captured the researchers' interest also in the area of ontologies. Ontologies are often represented in description logics (DL) that enable reasoning. The problem of abductive reasoning (Elsenbroich et al., 2006; Halland and Britz, 2012b), is highly relevant in real applications, such as ontology debugging, fault diagnosis, multi-media interpretation, manufacturing control.

An abduction problem is a pair $\mathcal{P} = (\mathcal{K}, O)$, where $\mathcal{K}$ is a knowledge base and O is a set of observations, s.t. $\mathcal{K} \not\models O$. The solution of $\mathcal{P}$ is an explanation $\mathcal{E}$, s.t. $\mathcal{K} \cup \mathcal{E}$ is consistent and $\mathcal{K} \cup \mathcal{E} \models O$. To filter undesired explanations, an explanation $\mathcal{E}$ is relevant ($\mathcal{E} \not\models O$), and subset minimal ($\mathcal{K} \cup \mathcal{E}' \not\models O$ for each $\mathcal{E}' \subsetneq \mathcal{E}$).

Consider the knowledge base $\mathcal{K}$:

$$\mathsf{Professor} \sqcup \mathsf{Scientist} \sqsubseteq \mathsf{Academician} \quad (1)$$

$$\mathsf{AssocProfessor} \sqsubseteq \mathsf{Professor} \quad (2)$$

and the observation $O = \{\mathsf{Academician(jack)}\}$. We are able to find following abductive explanations of $\mathcal{P} = (\mathcal{K}, O)$: $\mathcal{E}_1 = \{\mathsf{Professor(jack)}\}$, $\mathcal{E}_2 = \{\mathsf{Scientist(jack)}\}$, and $\mathcal{E}_3 = \{\mathsf{AssocProfessor(jack)}\}$.

2 Motivation

Halland and Britz (2012a) proposed an ABox abduction algorithm for DL $\mathcal{ALE}$ (Baader et al., 2003). In their work, they introduced a proposal of an algorithm based on Reiter's minimal hitting set algorithm. Their algorithm computes all models of the knowledge base in preprocessing using so called extended semantic tableau for DL. The motivation for this approach was to avoid overhead when compared to the existing translation-based solutions (Du et al., 2012; Klarman et al., 2011) and also to utilise optimization techniques for DL tableau algorithms. However, they did not formally prove soundness and completeness and did not provide an implementation, which opens the space for our work.

3 Goals

In our work, we focus on the development and the implementation of a tableau-based ABox abduction algorithm for DL, based on the Halland and Britz algorithm (Halland and Britz, 2012a), and on the Reiter's work (Reiter, 1987). Particularly, we focus on more expressive DLs, emphasizing flexibility, optimality, and effectivity. We study the theoretical properties of our proposal. We will also empirically evaluate our implementation and compare it with other approaches on experimental data.

4 Results

We have proposed an ABox algorithm for DL $\mathcal{ALCHO}$ (Baader et al., 2003) based on Halland and Britz work, that computes models of the knowledge base on-the-fly instead of computing them in the preprocessing.

Observation is in the form of a set consisting of ABox assertions, including role assertions and negated role assertions. Explanations are limited to a set of atomic concept and role assertions, and negated atomic concept and role assertions. Every explanation is explanatory, consistent, relevant, and subset minimal.

We have formally proved soundness and completeness of this algorithm with respect to the class of observations and explanations given above. The algorithm was implemented using the Pellet reasoner.

We have already published a use case about abductive reasoning in medical domain (Pukancová and Homola, 2015). The first proposal of the algorithm was also already published (Pukancová and Homola, 2016). Our latest results were submitted to the DL workshop 2017.

5 Future Work

In the future, we will consider optimization techniques that are not involved yet. The semantic minimality will be also considered (e.g. the explanation $\mathcal{E}_3$ in example above is not semantically minimal, as $\mathcal{E}_3 \cup \mathcal{K} \models \mathcal{E}_1$). In our opinion, in some abductive problems anonymous nodes may play role. We will investigated this cases and consider the extension of the algorithm.

References

Baader, F., Calvanese, D., McGuinness, D. L., Nardi, D., and Patel-Schneider, P. F., editors (2003). *The Description Logic Handbook: Theory, Implementation, and Applications*. Cambridge University Press.

Du, J., Qi, G., Shen, Y., and Pan, J. Z. (2012). Towards practical ABox abduction in large description logic ontologies. *Int. J. Semantic Web Inf. Syst.*, 8(2):1–33.

Elsenbroich, C., Kutz, O., and Sattler, U. (2006). A case for abductive reasoning over ontologies. In *Proceedings of the OWLED*06 Workshop on OWL: Experiences and Directions, Athens, Georgia, USA, November 10-11, 2006.*

Halland, K. and Britz, K. (2012a). Abox abduction in $\mathcal{ALC}$ using a DL tableau. In *2012 South African Institute of Computer Scientists and Information Technologists Conference, SAICSIT '12, Pretoria, South Africa, October 1-3, 2012*, pages 51–58.

Halland, K. and Britz, K. (2012b). Naïve ABox abduction in $\mathcal{ALC}$ using a DL tableau. In *Proceedings of the 2012 International Workshop on Description Logics, DL-2012, Rome, Italy, June 7-10, 2012*. Sun SITE Central Europe (CEUR).

Klarman, S., Endriss, U., and Schlobach, S. (2011). ABox abduction in the description logic $\mathcal{ALC}$. *Journal of Automated Reasoning*, 46(1):43–80.

Peirce, C. S. (1878). Deduction, induction, and hypothesis. *Popular science monthly*, 13:470–482.

Pukancová, J. and Homola, M. (2015). Abductive reasoning with description logics: Use case in medical diagnosis. In *Proceedings of the 28th International Workshop on Description Logics (DL 2015), Athens,Greece, June 7-10, 2015.*

Pukancová, J. and Homola, M. (2016). Tableau-based abox abduction for description logics: Preliminary report. In *Proceedings of the 29th International Workshop on Description Logics, Cape Town, South Africa, April 22-25, 2016.*

Reiter, R. (1987). A theory of diagnosis from first principles. *Artif. Intell.*, 32(1):57–95.

How to combine entailment and counterfactuals? Relevant logics and the problem of interpreting the sentences of the form $a \to b$

Aleksandra Samonek

March 8, 2017

Introduction

Although the symbol $\to$ is frequently used in philosophical logic, its interpretation varies depending on the underlying goal model. $\to$ may be defined to express either entailment or a counterfactual conditional. The formal interpretations of $\to$ often render the sentences of the form $a \to b$ incomparable in between different contexts.

The principal objective of my research project is to propose a unifying formal interpretation of the sentences of the form $a \to b$, that is an account that will satisfy the following requirements: (i) both contexts of application are considered, (ii) certain relevance criteria are met, and (iii) the underlying logic is not weakened.

The role of relevant logics

An entailment statement is of the form *a entails b*. Although classical logic provides sample valuation rules for an entailment of the form $a \to b$, the classical interpretation of entailment statements proved insufficient for the purposes of understanding the nature of entailment, especially the relationship between the precedent and the antecedent. A number of proposals for a more adequate interpretation originated from *relevant logics*, that is systems whose formalism allows only such arrow statements in which the premisses are in some predefined way relevant to the conclusion. Among many of the proponents of logical systems based on relevant entailment are [1, 2, 3, 4].

In my research I also consider counterfactual statements, that is statements of the form *if a had been true, then b*. Many theories of counterfactuals follow the classical criteria and fail to rid themselves of counterfactuals which result from classical consequence allowing for a *non sequitur*. Making use of the Routley-Meyer semantics for relevant logics and the notion of a conditional based on a selection function, Mares and Fuhrmann [5] proposed a relevant theory of conditionals. Mares argues that improving the theory of counterfactuals and including certain criteria for relevance will be to the advantage to a number of theories which utilize counterfactuals, e. g. Lewis' theory of causation and Chellas' dyadic deontic logic.

Goal directed approach to logic

I plan to make use of a goal directed approach to logic worked out by Diderik Batens, Dagmar Provijn [6, 7, 8] and Peter Verdée [9]. Unlike axiomatic or Fitch style proofs, in a goal directed proof one proves a candidate conclusion (called the goal of the proof) from a set of premisses by writing down lines starting with the goal and gradually justifying all elements of the goal (the subgoals), as far as possible, using the premisses. Lines of these proofs have a conditional character and have the following very helpful characteristic: the conditions of lines are always relevant for the consequent and *vice versa*. This characteristic holds even in case of goal directed proofs for classical logic.

Inge De Bal and Peter Verdée [10] devised a relevant logic NTR that explains the relevance characteristics of the lines of goal directed proofs. They have also developed an insightful diagrammatic proof method for NTR. These diagrams capture the relevant entailment relation in a classical logic context and so do not weaken classical logic.

References

1. Restall, G., 1996, "Information Flow and Relevant Logics," in J. Seligman and D. Westerstahl (eds.), Logic, Language and Computation (Volume 1), Stanford: CSLI Publications, pp. 463–478.
2. Fine, K., 1974, "Models for Entailment," Journal of Philosophical Logic, 3: 347–372.
3. Mares, E.D., 1997, "Relevant Logic and the Theory of Information," Synthese, 109: 345–360.
4. Wansing, H., 2002, "Diamonds are a Philosopher's Best Friends," Journal of Philosophical Logic, 31: 591–612.
5. Mares, E.D. and A. Fuhrmann, 1995, "A Relevant Theory of Conditionals," Journal of Philosophical Logic, 24: 645–665.
6. Batens, D., 2002, On a partial decision method for dynamic proofs. In "Paraconsistent Computational Logic", The papers presented at the ICLP 2002 workshop Paraconsistent Computational Logic, as part of the FLoC, pp. 91–108.
7. Batens, D., 2006, A diagrammatic proof search procedure as part of a formal approach to problem solving. In Magnani, L., editor, "Model-Based Reasoning in Science and Engineering", College Publications, pp. 2–265.
8. Batens, D. and Provijn, D., 2003, "Pushing the search paths in the proofs. a study in proof heuristics", Logique Et Analyse, 173:113–134.
9. Verdée, P., 2012, "A proof procedure for adaptive logics", Logic Journal of IGPL.
10. Verdée, P. and Bal, I. D., 2015, "A new approach to classical relevance", Studia Logica, 103(5):919–954.

Unpacking Broome's philosophical account of reasoning: a formal framework

A. Staras (Two-page Abstract)*

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Can rationality be reached by reasoning?

John Broome (2013), a philosopher and economist, has recently developed a philosophical *account of reasoning*. The account explores the relations of consequence that hold between attitudes (e.g. beliefs, intentions) and not between propositions or sentences as usual (2013, p. 254). This idea, Broome argues, captures the intuitive notion of reasoning as a mental activity conducted in a language through which you give rise to new attitudes from existing ones following a reasoning rule. The account starts with the primitive notion of mental states as attitudes towards particular propositions, and does not rely on absences of mental states (e.g. non-beliefs) (2013, p. 278). On this account, you can produce an attitude but cannot remove it by *explicit reasoning* (see also Wilson et al. (2000) for a relevant psychological account).

In this presentation I will try to unpack Broome's philosophical arguments that defend this account, and represent both rationality and reasoning in a simple formal model of mental states. I will then use this model to fit very basic requirements of theories of rationality (i.e. decision theory and philosophical logic) and investigate the limitations of Broome's account of reasoning relative to these requirements.

With mental states we want to model an agent who operates on meanings, not on symbols that represent meanings (see Broome 2013, p. 232, 253-4 and compare with Stenning and Van Lambalgen (2012) who develop a mental model of reasoning focusing on how the content of a reasoning process relates to descriptive and deontic attitudes). We shall say that an agent operates on the meanings if she operates on the content of both the proposition and the attitude, and uses a sentence to express *explicitly* these meanings. As a natural first step we present a simple model of mental states. In our model an agent operates *in* an *environment* in which:

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1. the agent forms mental states which are attitudes towards particular propositions
2. there are combinations of mental states which are allowed by rationality
3. the agent can change mental states by following rules of reasoning

Formally, the tuple $(\mathcal{L}, \mathcal{A}, \mathcal{T}, \mathcal{S})$ denotes the agent's environment where $\mathcal{L}$ contains the possible objects of attitudes, $\mathcal{A}$ the possible attitudes such as beliefs, intentions, and preferences, $\mathcal{T}$ captures which combinations of mental states are rationally allowed, and S captures how the agent can reason from some mental states to others. A mental state is a tuple $(p_1, p_2, ..., p_n, a)$, called the attitude a towards $p_1, p_2, ..., p_n$, where a is an attitude in $\mathcal{A}$, and $p_1, p_2, ..., p_n$ are propositions in $\mathcal{L}$. Each attitude comes with i) a number of places n in $\mathbb{N}^+ = \{1, 2, ...\}$, and ii) a domain of propositions $D \subseteq \mathcal{L}$. The number of places and the domain tell us that the attitude applies to combinations $p_1, p_2, ..., p_n$ of n propositions that belong to D.

Our notion of mental states applies to different theories in the sense that different theories of rationality recognise some specifications of attitudes and propositions. To demonstrate the model we consider theories of rationality from decision theory and philosophical logic and investigate the reasoning implicit in these theories while keeping (or weakening) their basic requirements. For example, we focus on two aspects that relate to decision theory. One aspect examines a very basic requirement of the theory; the relation between preferences and intentions to choose. The other aspect examines basic requirements on preferences (e.g. completeness and transitivity in Savage (1972)).

To do so, we classify rational requirements in a taxonomy of requirements typically found in modern logics and decision theoretic axiomatizations (e.g. transitivity, non-contradiction, completeness of preferences). The taxonomy consists of four types of requirements, that is two pairs of duals. By this we mean that the requirement R satisfied by the set of mental states C, is the dual of the requirement R' satisfied by C^* (= the set of mental states not in C). We characterize the taxonomy introducing the notions of exhaustiveness and exclusiveness varying the combinations of mental states. The taxonomy helps to build Broome's idea of satisfying a requirement by rules of reasoning with mental states that take the form *from... derive*. The upshot is that some requirements have corresponding rules, some do not. Those are not satisfied by Broome's explicit account of reasoning. We show that reasoning rules exists for specific types of requirements and do not exist for some other types of requirements.

References

Broome, John, *Rationality through reasoning*, John Wiley & Sons, 2013.

Savage, Leonard J, *The foundations of statistics*, Wiley, 1972.

Stenning, Keith and Michiel Van Lambalgen, *Human reasoning and cognitive science*, MIT Press, 2012.

Wilson, Timothy D, Samuel Lindsey, and Tonya Y Schooler, "A model of dual attitudes.," *Psychological review*, 2000, *107* (1), 101.

Analysing Ranking-based Semantics in Logic-Based Argumentation with Existential Rules.

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Abstract

Argumentation is a reasoning method in presence of inconsistencies that is based on the construction and evaluation of interacting arguments. In his seminal paper [5], Dung introduced the most abstract argumentation framework which consists of a set of arguments, a binary relation between arguments (called attack) and an *extension-based semantics* to extract subsets of arguments, representing consistent viewpoints, called *extensions*. Recently, another way of selecting some arguments was proposed: *ranking-based semantics*, which ranks arguments based on their controversy w.r.t. attacks [3], i.e. arguments that are attacked "more severely" are ranked lower than others. Extension-based semantics and ranking-based semantics are the two main approaches that I plan to focus on in my future works.

Logic-based argumentation [1] consists in instantiating argumentation framework with an inconsistent knowledge base expressed using a given logic that can be used in order to handle the underlying inconsistencies. It has been extensively studied and many frameworks have been proposed (assumption-based argumentation frameworks, DeLP, deductive argumentation or ASPIC/ASPIC+, etc.). In my current work, I chose to work with a logic that contains existential rules and to instantiate a deductive argumentation framework already available in the literature [4] with it. I made the choice of existential rules logic because of its expressivity and practical interest for the Semantic Web. Working with existential-rules instantiated argumentation frameworks is challenging because of the presence of special features (n-ary conflicts or existential variables in rules) and undecidability problems for query answering in certain cases.

The research question of my thesis is: ***"Can the gap between extension-based semantics and ranking-based semantics be bridged in the context of logic-based argumentation with existential rules ?"***

In a first work, I addressed the lack of consideration of the existing tools for handling existential rules with inconsistencies by introducing the first application workflow for reasoning with inconsistencies in the framework of existential rules using argumentation (i.e. instantiating ASPIC+ with existential rules [7]). The significance of the study was demonstrated by the equivalence of extension-based semantics outputs between the ASPIC+ instantiation and the one in [4].

Then, I focused on the practical generation of arguments from existential knowledge bases but soon realised that such a generating tool was nonexistent and that the current argumentation community did only possess randomly generated or very small argumentation graphs for benchmarking purposes. I thus created a tool that generates argumentation graphs from existential knowledge bases. This study was significant because I not only conducted a study of theoretical structural properties of the graphs induced by existential-rules-instantiated argumentation frameworks as defined in [4], but I

also analysed the behaviour of several solvers from an argumentation competition [9] regarding the generated graphs, and I studied whether their ranking (w.r.t. performance) was modified in the context of existential knowledge bases.

It is worth noticing that the number of arguments in [4] is exponential w.r.t. the size of the knowledge base. Thus, I extended the structure of arguments in [4] with minimality and studied notions of core [2] in order to reduce the size of the produced argumentation frameworks. What was surprising was that applying ranking-based semantics on a core of an argumentation framework gives different rankings than the rankings obtained from the original argumentation framework [8]. The salient point of this paper was the formal characterisation of these changes w.r.t. the proposed properties defined in [3].

In my first year of PhD, I obtained a better understanding of both extension-based semantics and ranking-based semantics in the particular framework of logic-based argumentation with existential rules.

In the next two years, I plan to first study the following question: "In which case does the output of a ranking-based semantics partially correspond to the output of an extension-based semantics ?" Indeed, since ranking-based semantics are generally easy to compute and extension-based semantics are hard (skeptical reasoning under the preferred semantics is located on the second level of the polynomial hierarchy), this work would be significant as it would be the first approximation of extension-based semantics. Moreover, since the output of extension-based semantics is conserved in the cores [2], it would similarly be interesting to find ranking-based semantics that keep the same outputs. Finally, I plan to export all of my results and apply them on previously studied real world use-cases obtained in the framework of the agronomy Pack4Fresh project [6].

References

[1] Leila Amgoud and Philippe Besnard. Bridging the Gap between Abstract Argumentation Systems and Logic. In *SUM 2009. Proceedings*, pages 12–27, 2009.

[2] Leila Amgoud, Philippe Besnard, and Srdjan Vesic. Equivalence in logic-based argumentation. *Journal of Applied Non-Classical Logics*, 24(3):181–208, 2014.

[3] Elise Bonzon, Jérôme Delobelle, Sébastien Konieczny, and Nicolas Maudet. A Comparative Study of Ranking-Based Semantics for Abstract Argumentation. In *Proceedings of the Thirtieth AAAI Conference on Artificial Intelligence, February 12-17, 2016, Phoenix, Arizona, USA.*, pages 914–920, 2016.

[4] Madalina Croitoru and Srdjan Vesic. What Can Argumentation Do for Inconsistent Ontology Query Answering? In *Scalable Uncertainty Management - 7th International Conference, SUM 2013, Washington, DC, USA, September 16-18, 2013. Proceedings*, pages 15–29, 2013.

[5] Phan Minh Dung. On the Acceptability of Arguments and its Fundamental Role in Nonmonotonic Reasoning, Logic Programming and n-Person Games. *Artif. Intell.*, 77(2):321–358, 1995.

[6] Bruno Yun, Pierre Bisquert, Patrice Buche, and Madalina Croitoru. Arguing About End-of-Life of Packagings: Preferences to the Rescue. In *Metadata and Semantics Research - 10th International Conference, MTSR 2016, Göttingen, Germany, November 22-25, 2016, Proceedings*, pages 119–131, 2016.

[7] Bruno Yun and Madalina Croitoru. An Argumentation Workflow for Reasoning in Ontology Based Data Access. In *Computational Models of Argument - Proceedings of COMMA 2016, Potsdam, Germany, 12-16 September, 2016.*, pages 61–68, 2016.

[8] Bruno Yun, Madalina Croitoru, and Pierre Bisquert. Are ranking semantics sensitive to the notion of core? In *Autonomous Agents and Multiagent Systems - Proceedings of AAMAS 2017, Sao Paulo, Bresil, 8-12 May, 2017*, 2017.

[9] Bruno Yun, Srdjan Vesic, Madalina Croitoru, Pierre Bisquert, and Rallou Thomopoulos. A structural benchmark for logical argumentation frameworks. In *IJCAI 2017, submitted*, 2017.